Standard Deviation

Lecture 18
Section 5.3.4

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Outline

1. Variability
2. The Standard Deviation
   - Examples
   - Alternate Formula
3. TI-83 Standard Deviations
4. Interpreting the Standard Deviation
5. Assignment

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- What do we mean by "variability" in the population?
- How do we measure it?
- The IQR is one way.
- We will learn another way.

An Example

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- For which date should you choose to predict the high temperature?
- On which date is the high temperature less variable?
- Naturally, you should choose the date with less variability.

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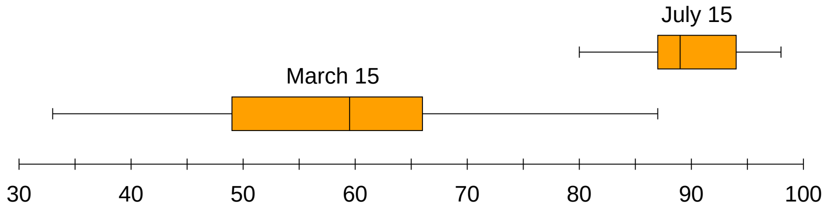

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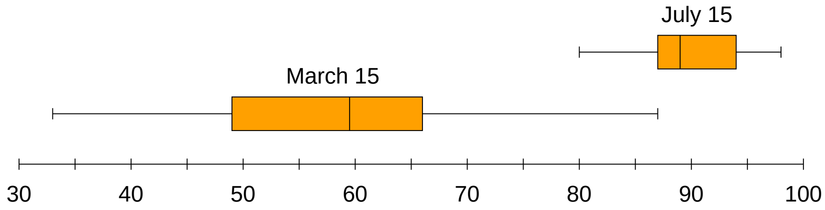

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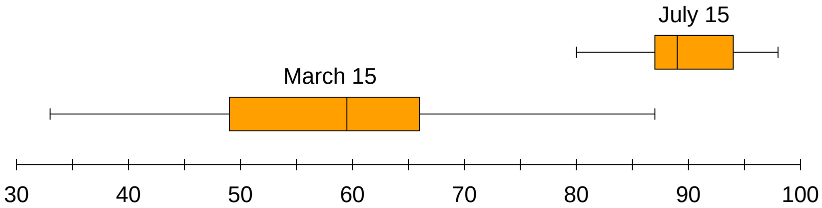


- Which is more variable?
- How much more variable is it?

Measures of Variability

- The IQR is one measure of variability.
  - IQR for March 15 data $= 66 - 49 = 17$.
  - IQR for July 15 data $= 94 - 87 = 7$.
- By that measure, the July temperatures are about $2\frac{1}{2}$ times as variable as the March temperatures.
- We will also measure variability by using
  - the variance.
  - the standard deviation.

Deviations from the Mean

Definition (Deviation)

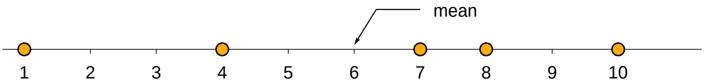
The deviation of an observation x is the difference between x and the sample mean $\overline{x}$.

$$\text{deviation of } x = x - \overline{x}.$$

For a member of the population, the deviation is measured from the population mean:

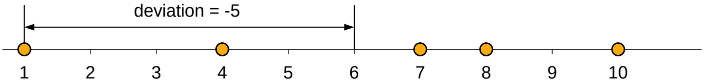
$$\text{deviation of } x = x - \mu.$$

Deviations from the Mean



Deviations from the mean

Deviations from the Mean



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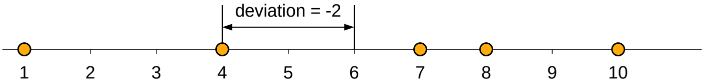

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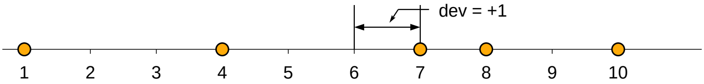

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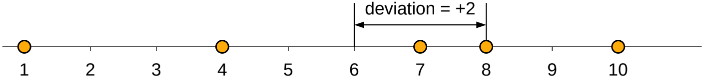

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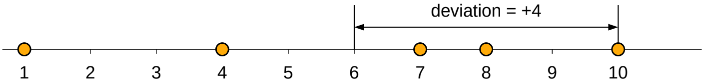

Deviations from the mean

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Deviations from the Mean

- How do we obtain *one* number that is representative of the whole set of individual deviations?
- Normally we use an average to summarize a set of numbers.
- Why will the average not work in this case?

Deviations from the Mean

- How do we obtain *one* number that is representative of the whole set of individual deviations?
- Normally we use an average to summarize a set of numbers.
- Why will the average not work in this case?
- It will not work because

$$\sum (x - \overline{x}) = 0.$$

Sum of Squared Deviations

- Rather than average the deviations, we will average their squares. That way, there will be no canceling.
- So we compute first the sum of the squared deviations.

Definition (Sum of squared deviations)

The sum of squared deviations, denoted SSX, of a set of numbers is the sum of the squares of their deviations from their mean.

$$\text{SSX} = \sum (x - \overline{x})^2.$$

Sum of Squared Deviations

Example (Calculating SSX)

- Let the sample be $\{1, 4, 7, 8, 10\}$.
- Then

$$\begin{aligned} \text{SSX} &= (1-6)^2 + (4-6)^2 + (7-6)^2 + (8-6)^2 + (10-6)^2 \\ &= (-5)^2 + (-2)^2 + 1^2 + 2^2 + 4^2 \\ &= 25 + 4 + 1 + 4 + 16 \\ &= 50. \end{aligned}$$

Sum of Squared Deviations

Practice

- Let the sample be $\{1, 5, 6, 9, 14\}$.
- Calculate
  - The sample mean.
  - The deviations.
  - The squared deviations.
  - The sum of the squared deviations.

Measuring Population Variability

Definition (Variance of a population)

The variance of a population, denoted σ^2, is the average of the squared deviations of the members of the population.

$$\sigma^2 = \frac{\sum (x - \mu)^2}{N}.$$

Definition (Standard deviation of a population)

The standard deviation of a population, denoted σ, is the square root of the population variance.

$$\sigma = \sqrt{\frac{\sum (x - \mu)^2}{N}}.$$

Estimating Population Variability

Definition (Variance of a sample)

The variance of a sample, denoted s^2, is the sum of the squared deviations of the members of the sample, divided by 1 less than the sample size.

$$s^2 = \frac{\sum (x - \overline{x})^2}{n - 1}.$$

Definition (Standard deviation of a sample)

The standard deviation of a sample, denoted s, is the square root of the sample variance.

$$s = \sqrt{\frac{\sum (x - \overline{x})^2}{n - 1}}.$$

The Sample Variance

- Theory shows that if we divided $\sum(x - \overline{x})^2$ by n, then s^2 would systematically underestimate σ^2.
- Theory also shows that if we divided $\sum(x - \overline{x})^2$ by $n - 1$ instead of n, then s^2 would not systematically underestimate nor overestimate σ^2.
- Therefore, we divide by $n - 1$.

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Example

Example (Calculating s)

- For the sample $\{1, 4, 7, 8, 10\}$, we found that

$$\text{SSX} = 50.$$

- Therefore,

$$s^2 = \frac{50}{4} = 12.5$$

and so

$$s = \sqrt{12.5} = 3.54.$$

Sum of Squared Deviations

Practice

- Let the sample be $\{1, 5, 6, 9, 14\}$.
- Calculate s^2 and s.

Example

- How does s compare to the individual deviations?
- We will interpret s as being "representative" of the deviations in the sample.
- Does that seem reasonable for the previous examples?

Example

- It turns out that the standard deviation of the March 15 high temperatures is 12.7.
- The standard deviation of the July 15 high temperatures is 4.5.

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Alternate Formula for SSX

- An alternate formula for SSX is

$$\text{SSX} = \sum x^2 - \frac{\left(\sum x\right)^2}{n}.$$

- Then, as before

$$s^2 = \frac{\text{SSX}}{n-1}$$

and

$$s = \sqrt{\frac{\text{SSX}}{n-1}}.$$

- Often this formula is easier to use.

Example

Example (Alternate formula for SSX)

- Let the sample be $\{-8, -4, 1, 3, 10\}$.
- Then
$$\sum x = 2$$
and
$$\sum x^2 = 64 + 16 + 1 + 9 + 100 = 190.$$
- So
$$\begin{aligned} \text{SSX} &= 190 - \frac{2^2}{5} \\ &= 189.2 \end{aligned}$$
- Then $s = \sqrt{\frac{189.2}{4}} = \sqrt{47.3} = 6.877$.

Sum of Squared Deviations

Practice

- Let the sample be $\{1, 5, 6, 7, 10\}$.
- Find $\sum x$.
- Find $\sum x^2$.
- Use the alternate formula to find SSX, s^2, and s.

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TI-83 - Standard Deviations

TI-83 Standard Deviations

- Follow the procedure for computing the mean.
- The display shows $\mathtt{Sx}$ and $\sigma\mathtt{x}$.
- $\mathtt{Sx}$ is the sample standard deviation.
- $\sigma\mathtt{x}$ is the population standard deviation.

Example

Example

TI-83 Standard Deviations

- Let the sample be $\{-8, -4, 1, 3, 10\}$.
- We get
  - $\mathrm{Sx} = 6.877$.
  - $\sigma\mathrm{x} = 6.151$.

Example

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TI-83 Standard Deviations

- Let the sample be $\{-8, -4, 1, 3, 10\}$.
- We get
  - $\mathrm{Sx} = 6.877$.
  - $\sigma\mathrm{x} = 6.151$.
- Which one is *s*?

Example

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TI-83 Standard Deviations

- Let the sample be $\{-8, -4, 1, 3, 10\}$.
- We get
  - $\mathtt{Sx} = 6.877$.
  - $\sigma\mathtt{x} = 6.151$.
- Which one is s?
- Then what is $\sigma\mathtt{x}$?

Sum of Squared Deviations

Practice

- Let the sample be $\{1, 5, 6, 7, 10\}$.
- Use the TI-83 to find s and s^2.
- What are the values of $\sum x$ and $\sum x^2$?

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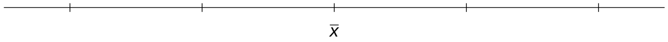
Interpreting the Standard Deviation

- Observations that deviate from $\overline{x}$ by much more than s are unusually far from the mean.
- Observations that deviate from $\overline{x}$ by much less than s are unusually close to the mean.

Interpreting the Standard Deviation



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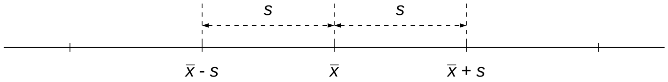

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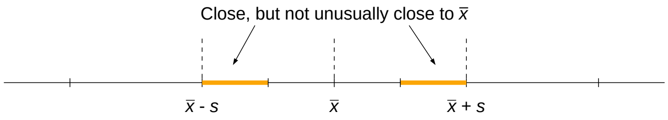

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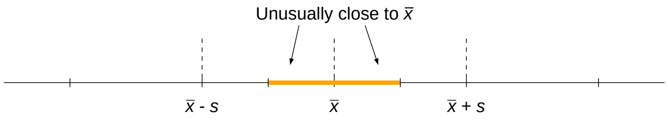

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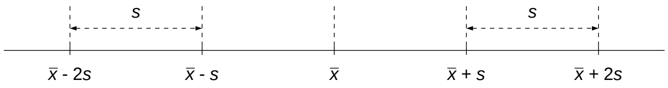

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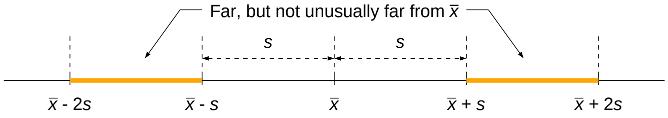

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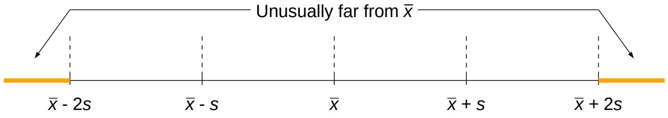

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Homework

- Read Section 5.3.4, pages 326 - 333.
- Let's Do It! 5.13, 5.14, 5.15.
- Page 333, exercises 10, 11, 14, 16 - 18, 20, 21.
- Chapter 5 review, p. 345, exercises 29 - 32, 36 - 40, 42 - 44, 47, 52, 53, 55.